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In like manner it may be shown that the average length of BP is

$$\frac{1}{3b}\Big(a^2\cos C+c^2\cos A-ac\sin A\sin C\log[\tan\tfrac{1}{2}A\tan\tfrac{1}{2}C]; \text{ and of } CP,$$

$$\frac{1}{3c}\Big(a^2\cos B+b^2\cos A-ab\sin A\sin B\log[\tan\tfrac{1}{2}A\tan\tfrac{1}{2}B].$$

Hence the required average is

$$M=\tfrac{1}{3}\Big[\Big(\frac{b^2}{c}+\frac{c^2}{b}\Big)\cos A+\Big(\frac{a^2}{c}+\frac{c^2}{a}\Big)\cos B+\Big(\frac{a^2}{b}+\frac{b^2}{a}\Big)\cos C-a\sin\tfrac{1}{2}A[b\sin B$$

$$+c\sin C]\log\tan\tfrac{1}{2}A-b\sin B[a\sin A+c\sin C]\log\tan\tfrac{1}{2}B$$

$$-c\sin C[a\sin A+b\sin B]\log\tan\tfrac{1}{2}C.$$

184. Proposed by HENRY HEATON, Belfield, N. D.

Through every point of the sides of a given square, straight lines are drawn across the square in every possible direction. What is their average length?

I. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

The problem evidently wants the average length of all lines terminated in opposite sides; otherwise the problem is the same as problem 169, three solutions of which have already been published.

Let $[a, x]$ be the coordinates of one end of the line, $[0, y]$ the coordinates of the other end. $\triangle$=the average length required.

$$\therefore \triangle=\frac{\int_0^a\int_0^x\sqrt{\{a^2+[x-y]^2\}}dx\,dy}{\int_0^a\int_0^x dxdy}, =\frac{2}{a^2}\int_0^a\int_0^x\sqrt{\{a^2+[x-y]^2\}}dxdy$$

$$=\frac{1}{a^2}\int_0^a\Big(x\sqrt{[a^2+x^2]}+a^2\log\frac{x+\sqrt{[a^2+x^2]}}{a}\Big)dx=a\{\tfrac{2}{3}[1-\sqrt{2}]+\log[1+\sqrt{2}]\}.$$

II. Solution by HENRY HEATON, Belfield, N. D.

Let P be a point in AB, PE a line perpendicular to AB, and PF a line drawn across the $\triangle PAD$. Put $AP=x$, and $\angle FPA=\theta$. Then supposing x constant, the average length of the lines drawn from P across the triangle PAD is

$$\int_0^{\tan^{-1}(a/x)}x\sec\theta\,d\theta\div\int_0^{\tan^{-1}(a/x)}d\theta=x\log\Big(\frac{\sqrt{[a^2+x^2]}+a}{x}\Big)\div\tan^{-1}[a/x].$$

Similarly the average length of lines drawn from P across the $\triangle PED$ is

$$a\log\left(\frac{\sqrt{[a^2+x^2]}+x}{a}\right)\div\tan^{-1}[x/a].$$

Hence the average length of lines drawn from P across the rectangle $PEDA$ is

$$\frac{2}{\pi}\left[x\log\left(\frac{\sqrt{[a^2+x^2]}+a}{x}\right)+a\log\left(\frac{\sqrt{[a^2+x^2]}+x}{a}\right)\right].$$

Hence the required average is

$$M=\frac{4}{a\pi}\int_0^a\left[x\log\left(\frac{\sqrt{[a^2+x^2]}+a}{x}\right)+a\log\left(\frac{\sqrt{[a^2+x^2]}+x}{a}\right)\right]dx$$

$$=\frac{6a}{\pi}\log[\sqrt{2}+1]-\frac{2}{\pi}\int_0^a\frac{xdx}{\sqrt{[a^2+x^2]}}=\frac{2a}{\pi}\{3\log\sqrt{[2}+1]-\sqrt{[2}+1]\}.$$

NOTE.—These solutions differ because both problems are stated in the indefinite form and the authors have assumed different laws of distribution. ED. F.

MISCELLANEOUS.

168. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Sum to n terms, $\sin\alpha\sin\beta+\sin\alpha-\beta\sin\beta+\gamma+\sin\alpha-2\beta\sin\beta+2\gamma+\ldots$

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

The series intended is evidently

$$\sin\alpha\sin\beta+\sin[\alpha-\beta]\sin[\beta+\gamma]+\sin[\alpha-2\beta]\sin[\beta+2\gamma]+\ldots\text{ to } n \text{ terms.}$$

Since $2\sin\alpha\sin\beta=\cos[\alpha-\beta]-\cos[\alpha+\beta]$ we get, if S=the sum

$$S=\tfrac{1}{2}\{\cos[\alpha-\beta]+\cos[\alpha-2\beta-\gamma]+\cos[\alpha-3\beta-2\gamma]+\ldots\text{ to } n \text{ terms}\}$$
$$-\tfrac{1}{2}\{\cos[\alpha+\beta]+\cos[\alpha+\gamma]+\cos[\alpha-\beta+\gamma]+\ldots\text{ to } n \text{ terms}\}.$$

Let $\alpha+\beta=\theta$, $=[\beta+\gamma]=\phi$, $[\alpha+\beta]=\psi$, $-[\beta-\gamma]=\rho$.

$$\therefore S=\tfrac{1}{2}\{\cos\theta+\cos[\theta+\phi]+\cos[\theta+2\phi]+\ldots+\text{ to } n \text{ terms}\}$$
$$-\tfrac{1}{2}\{\cos\psi+\cos[\psi+\rho]+\cos[\psi+2\rho]+\ldots+\text{ to } n \text{ terms}\}.$$